INTEREST RATES ON THE RISE

Washington (AP) The Federal Reserve announced yesterday that interest rates will be on the rise, up by about one quarter of one percent. This is the third such increase in three months. Greenspan, who indicated the main factor for the increase was to put a control on the housing market, which has been slowly rising. Back in March of this year the Federal Reserve increased rates across the board to control the average mortgage rate.

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As this issue of Peer Review goes to press, America is entering the final throes of the 2004 presidential campaign. Accordingly, we are beset by public opinion polls and by data-driven arguments about the relative merits of various policy alternatives. Our choices as voters may depend, in part, on our ability to sort through competing claims rooted in quantitative measures. Yet while its direct relevance to democracy may be more readily discernible in an election year, quantitative literacy—or “numercy,” as it is sometimes called—has become increasingly important to citizenship more generally. Indeed, effective participation in civic life depends more than ever upon one’s ability to understand quantitative information and to make informed decisions based upon it.

Citizens are regularly confronted with a dizzying array of numerical information. On a given day, for example, the media may report changes in the consumer price index or federal interest rates, results of clinical trials, statistics from an educational assessment of local schools, findings from a study of the long-term health effects of a widely used product; the list could go on almost endlessly. Moreover, near-omnipresent computers generate—and the Internet makes available—a staggering amount of information, much of it quantitative.

For a quantitatively literate citizen, access to this wealth of information is potentially empowering. The reverse also is true, however. A quantitatively illiterate citizen—one who is unable to evaluate statistical arguments competently, for example, or incapable of grasping the potential implications of data trends—may be easily mystified. As Lynn Steen has put it, “an innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg’s time.”

As the economic and social effects of globalization continue to transform our notions of citizenship and broaden the opportunities and the need for responsible engagement, colleges and universities are working to strengthen the historical link between liberal education and civic engagement. There is also still much work to be done to help students see the important links between what they learn in college and their current and future lives as citizens of a diverse and globally-interconnected society.

Understanding the relationship between quantitative literacy and mathematics is a vital first step for curricular reform efforts, for efforts to inform students about important outcomes of college, and for attempts to clarify definitions of college readiness. Quantitative literacy is not a watered-down version of mathematics, and neither is it a replacement for mathematics; students need both. And as with writing or critical thinking skills, responsibility for helping students to develop quantitative literacy must be shared broadly across the curriculum.

This issue of Peer Review offers a primer on quantitative literacy in college today. In these pages, among other things, you’ll find definitions, arguments for the importance of quantitative literacy, discussion of its relationship to mathematics, and different models for developing it. And yes, you’ll even find some data, statistics, and other quantitative information about it.

—DAVID TRITELLI
Everything I Needed to Know about Averages . . . I Learned in College

By Lynn Arthur Steen, professor of mathematics, St. Olaf College

Several months ago, the conservative-leaning American Council of Trustees and Alumni (ACTA) excoriated America’s leading colleges and universities with a report documenting the “failure of general education” (ACTA 2004). Among many cited shortcomings, one—emphasized in bold face in the opening paragraph—is that “mathematics is no longer required at 62% of the examined institutions.”

Much could be said about the educational merits of traditional core curricula or the political agendas served by debates about the core. But that is not what I found most interesting about this report. Rather, it was the messages hidden in the fine print. There, in the endnotes, lie intriguing clues about collegiate mathematics—both about its place in general education and its role in the ACTA study.

First, many colleges and universities call this core requirement not “mathematics” but “quantitative reasoning,” although variations abound: “quantitative or formal reasoning,” “mathematical thinking,” “mathematical and logical analysis,” “quantitative and deductive sciences,” “formal reasoning and analysis,” or “quantitative and deductive reasoning.” All of these stress the processes of mathematics (reasoning, deduction, analysis) rather than its components (algebra, geometry, statistics, calculus).

Second, these requirements are often fulfilled with courses that help students build connections between mathematics and other subjects, courses that reveal how quantitative reasoning is used across the entire spectrum of collegiate studies:

- Counting People;
- Economics and the Environment;
- Health Economics;
- Introduction to Energy Sources;
- Introduction to Population Studies;
- Language and Formal Reasoning;
- Limnology: Freshwater Ecology;
- Maps, Visualization, and Geographical Reasoning;
- Practical Physics: How Things Work;
- Quantifying Judgments of Human Behavior.

Here’s what caught my attention: in every case where colleges allowed students to fulfill a quantitative reasoning requirement with courses such as these, the ACTA study judged the institutions as not including “mathematics” in its core curriculum. These colleges wound up on the 62 percent blacklist. But colleges that required a course in college algebra—whose pièce de résistance is the manipulation of negative fractional exponents—were checked off for having a suitable “mathematics” core requirement.

Quantitative Literacy

This ACTA analysis demonstrates the presence of “two mathematics” (see Bernard Madison’s article in this issue). One is an abstract, deductive discipline created by the Greeks, refined through the centuries, and employed in every corner of science, technology, and engineering. The other is a practical, robust habit of mind anchored in data, nourished by computers, and employed in every aspect of an alert, informed life. This is what these many colleges call “quantitative reasoning,” what many other countries
call “numeracy,” or what I’ll call “quantitative literacy” (or QL for short).

Although clearly related, quantitative literacy and mathematics are not the same. Whereas mathematics rises above context, QL is anchored in context. Whereas the objects of mathematical study are ideals (in the Platonic sense), the objects of QL are data, generally measurements retrieved from some computer’s data warehouse. Because quantitative reasoning relies on concepts first introduced in middle school—averages, percentages, graphs—many believe that QL is just watered down mathematics (and thus should not satisfy a “mathematics” requirement). Some academics, typically mathematicians, argue that students should complete QL by the end of high school; in this view, it is not a central (or even proper) responsibility of higher education. Others, typically not mathematicians, argue that QL is too important to be left to mathematicians, whose training inclines them more toward Platonism than earthly practicality.

The issue of the core curriculum raised in the ACTA study is exactly the central issue for quantitative literacy. Whereas, typically, college-level mathematics serves primarily preprofessional purposes (as prerequisites for particular courses), quantitative literacy is essential for all graduates’ personal and civic responsibilities. College-level quantitative literacy is inextricably connected to virtually all areas of undergraduate study.

Understanding compound interest is a trite staple of QL expectations, but it is nonetheless a good example whose significance is not truly manifest until students are of college age. Only when students become responsible for their own loans do the formal calculations they may have learned in eighth grade become personally meaningful. (Few adults realize the extraordinary difference even a quarter-percent change in interest rates can make on payoff time for a fixed payment loan.) More generally, it is in college where many students study historical events and first become personally engaged in social and political causes whose roots often lie just beneath the surface in the financial conditions of individuals or states. The habit of thinking quantitatively—even more, of seeking quantitative evidence—requires repeated practice in many different contexts. For that reason, many colleges have replaced course requirements (whether in mathematics or QL) with programs of “QL across the curriculum.”

Less obvious, perhaps, than compound interest are the many examples of public policy issues requiring voters’ attention that depend significantly on subtle quantitative reasoning. I’m not referring to obvious, although nonetheless complex, issues such as projecting future deficits or counting votes accurately, but to situations where quantitative traps lie hidden beneath routine calculations of percentages and averages. I offer a few examples from issues in public education; similar examples abound in every area of public policy.

**Percentages**

Major problems beset public education, leading to significant gaps in performance and to high dropout rates. Measuring the gap between expectations and accomplishment is a complex, multidimensional challenge that every parent recognizes as a task requiring judgment and interpretation. But measuring dropout rates seems simple: just apply the formula for percentages everyone learned in the seventh grade. Here’s one result, as reported by the New York Times on August 13, 2003, under the headline “The ‘Zero Dropout’ Miracle” (Winerip 2003):

Robert Kimball, an assistant principal at Sharpstown High School [in Houston], sat smack in the middle of the “Texas miracle.” His poor, mostly minority high school of 1,650 students had a freshman class of 1,000 that dwindled to fewer than 300 students by senior year. And yet—and this is the miracle—not one dropout to report!

Nor was zero an unusual dropout rate in this school district that both President Bush and Secretary of Education Rod Paige have held up as the national showcase for accountability and the model for the federal No Child Left Behind law. Westside High here had 2,308 students and no reported dropouts; Wheatley High 731 students, no dropouts. A dozen of the city’s poorest schools reported dropout rates under 1 percent.

Now, Dr. Kimball has witnessed many amazing things in his 58 years. Before he was an educator, he spent 24 years in the Army, fighting in Vietnam, rising to the rank of lieutenant colonel and touring the world. But never had he seen an urban high school with no dropouts. “Impossible,” he said. “Someone will get pregnant, go to jail, get killed.” Elsewhere in the nation, urban high schools report dropout rates of 20 percent to 40 percent.
A miracle? “A fantasy land,” said Dr. Kimball. “They want the data to look wonderful and exciting. They don’t tell you how to do it; they just say, ‘Do it.’”

As it turns out, there are a number of different ways to “do it,” each with its own justification. Finding one that produces the desired answer of zero may take some effort, but it is not beyond the realm of plausible quantitative argument. One simple way is to divide the number of high school graduates by the number of entering freshmen four years earlier, and then to subtract from 100 percent. If a high school is growing, it is not unreasonable that this dropout calculation yields a number close to zero. Another approach—the one used in higher education—tracks a specific entering cohort of students through their four years of high school, ignoring all other students in the school (e.g., transfers). A third common method is to classify the reasons students leave school each year (transfer, work, jail, death, dropout, etc.) and then report only the “dropout” classifications.

Each method has distinct characteristics that may make it more or less useful for a particular purpose. The first and simplest calculation is highly sensitive to irrelevant circumstances such as growth and transfers. The second, being limited to a subset of students, may not represent the quality of education received by all students. The third attempts to account for why students leave a school, thereby limiting the meaning of “dropout” to students for whom no other reason may apply. (Setting aside the possibility of deliberate misrepresentation, this may explain the Texas miracle: “Do it” can be taken by teachers as a challenge to find any reason other than “drop out” to explain why students left school.)

In the seventh grade students might be asked, “if 500 students enter Abraham

In 1995, the Science Advisory Committee at The College Board began asking whether questions on important science tests such as AP Biology and AP Chemistry adequately reflected the increasingly quantitative nature of these sciences. Prompted by this request, Robert Orrill, then director of the Office of Academic Affairs at The College Board, began a series of studies into the nature and educational role of quantitative literacy. These studies quickly expanded well beyond the foundations of science into issues of citizenship, economics, and democracy itself. Within a few years, they became the central focus of the new National Council on Education and the Disciplines (NCED) at the Woodrow Wilson National Fellowship Foundation.

Joining this effort from different perspectives were the Mathematical Sciences Education Board (MSEB) at the National Research Council (NRC) and the Mathematical Association of America (MAA), which represents nearly 30,000 faculty who teach college-level mathematics. In 1996, an MAA committee had prepared a report on quantitative reasoning for college graduates from the perspective of mathematicians. In 2001 these three organizations jointly organized a forum at the National Academy of Sciences on the theme “Quantitative Literacy: Why Numeracy Matters for Schools and Colleges.” Simultaneously, the MAA’s Committee on the Undergraduate Program in Mathematics (CUPM) undertook a major review of collegiate mathematics that contains many “QL-friendly” suggestions. A companion report records the mathematical and quantitative expectations of eighteen partner disciplines, providing a virtual map of quantitative literacy.

Six reports emerged from these studies that deal, on the one hand, with the challenge of quantitative literacy and, on the other hand, with the challenges facing undergraduate mathematics that overlap but are by no means the same as those of QL:

Lincoln High School as freshmen and 400 graduate, how many dropped out?” The next time they may be asked to consider what “drop out” means is when they vote for school board candidates or on a school levy referendum. Unless their quantitative literacy has been significantly enhanced, citizens are likely to enter the voting booth with a seventh-grade concept of dropout rate. That’s why courses such as those discounted by the ACTA study are so important: students who took a course on, say, “Economics of Education” would be far better equipped to fulfill their responsibilities as educated citizens than those who met their mathematics requirement by simplifying rational functions in a college algebra course.

Averages

Students in my hypothetical “Economics of Education” course are likely to learn that averages, like percentages, are also a source of mysteries. A recent study shows that the average verbal SAT score did not improve during the two decades between 1981 and 2002 (Bracey 2004). But during that same period, the average scores of each of the six major ethnic categories used in reporting SAT data (white, black, Asian, Puerto Rican, Mexican, and American Indian) increased by amounts ranging from eight to twenty-seven points. Yet the overall average did not budge—enabling skeptics to claim that all the money invested in education during the last two decades has produced no noticeable improvement.

A quantitatively literate college graduate would recognize this mystery as a classic example of Simpson’s Paradox: changes in composition can cause the whole to show trends opposite to each of its parts when considered separately. Demagogues rely on the public’s simplistic seventh-grade understanding of how numbers work to ply their trade. But in today’s data-drenched society,


The interest in QL evidenced by these publications has generated QL workshops for faculty in a variety of disciplines as well as curricular projects on several campuses (e.g., Colorado College, Dartmouth College, the Evergreen State College, Hollins University, Lawrence University, Macalester College, Trinity College, University of Nevada-Reno). Several organizations—some brand new—have been established to assist individuals, departments, and campuses in creating effective QL programs. For example, the Northeast Consortium for QL holds meetings each spring for faculty and staff associated with the “QL Centers” that operate at many colleges in New England. The MAA recently established a special interest group on quantitative literacy.2 Newest of all, a National Numeracy Network3—one of several NCED initiatives—connects several institutions with major QL projects.

Further References


1. www.maa.org/past/ql/ql_toc.html
2. www.css.tayloru.edu/~mdelong/qslsigmaa/frames.html
3. www.math.dartmouth.edu/~mqed/index.html
sometimes no one really understands what is going on.

Society’s reliance on data as a justification for decisions increased gradually throughout the nineteenth and twentieth centuries (Desrosières 1998; Porter 1995), but it has taken a significant leap during the last two decades—the chief reason being the vast quantity of data that computers dispose. More recently, the importance of QL (and the consequences of quantitative illiteracy) has been greatly magnified, if not totally transformed, by the behavior of computer networks. Recent financial scandals, for example, were enabled by clever bookkeeping that displayed apparent corporate gains while every part of the business was actually losing money. Yet even professionals well aware of Simpson’s Paradox did not detect these machinations.

Something deeper than just clever or illegal accounting seems to be at work. Two years ago, an analysis of this new “culture of finance” was presented at the International Congress of Mathematicians and subsequently published by the American Mathematical Society (Poovey 2003). It suggests that the invisible impact of “mathematical abstractions” on modern society has generated a “new form of governance.” In short, the mechanisms of quantification that began with averages and percentages have become just as abstract—and hence as powerful—as mathematics itself. We just haven’t realized it yet.

**QL on Campus**

My main point in these examples is not to argue that QL is important; I’ve rarely met anyone who doubts that. Rather, my point is that QL is sufficiently sophisticated to warrant inclusion in college study and, more important, that without it students cannot intelligently achieve major goals of college education. Quantitative literacy is not just a set of precollege skills. It is as important, as complex, and as fundamental as the more traditional branches of mathematics. Indeed, QL interacts with the core substance of liberal education every bit as much as the other two R’s, reading and writing.

Quantitative literacy differs from mathematics primarily by being anchored in real contexts. While this anchor is generally a source of strength—notably for improved student motivation and learning—it is also a source of structural weakness. Since QL is not a discipline in the traditional sense, it lacks the academic infrastructure of departments, journals, and professional associations. By its nature, QL is dispersed and, thus, almost invisible. Many efforts are now underway to make QL visible and to establish a strong presence in the ecology of liberal education. Some are described in the box on pages 6–7, others later in this issue.

From all these sources one clear priority has emerged: the need to develop benchmarks for quantitative literacy that can guide both curriculum and assessment in grades 10-16. Since QL is relatively new and since it lives in the matrix of other disciplines, neither higher education professionals nor public leaders have a clear understanding of suitable performance expectations. Consensus on expectations is a desirable (but not inevitable) outcome of various approaches to mathematical and quantitative literacy in core curricula and, more broadly, general education. This issue of Peer Review is an important step in the process of building consensus.

**References**


In his classic essay on the aesthetics of mathematics, *A Mathematician’s Apology* (1940), the prominent British mathematician G. H. Hardy identifies two mathematics: real mathematics and “trivial” (or useful) mathematics. Although writing from a rarefied point of view, Hardy aptly describes a disparity that has existed for centuries, one that persists in twenty-first-century America as a division between the rigorous mathematics that real mathematicians study, appreciate, and extend and the contextualized mathematics of everyday life.

The historian Patricia Cohen confirms a similar division, in colonial times, between commercial arithmetic and more sophisticated mathematics. Fearing that bourgeois lads would find mathematics too difficult, colonial and precolonial textbook writers attempted to strip arithmetic to its essentials. But, as Cohen points out (1982, 27-28), “in fact they cut it into incoherent bits and made it an arcane subject, almost impossible to learn.” To make matters worse, arithmetic was suffused with commercial meanings, which caused those not destined to a life in commerce to learn no arithmetic at all. The endemic lack of arithmetic skill among tradesmen in the seventeenth and eighteenth centuries is indicated by the popularity of “ready reckoners,” books containing page after page of tables showing multiples of the unit costs of common commodities at a variety of prices.

As American democracy has developed, the quantitative demands on its citizens have grown; indeed, largely driven by the power of computers to amass and analyze data, these demands have exploded over the past two decades. The ability to deal with the quantitative demands of everyday life is what we call quantitative literacy (QL for short). Robert Orrill, director of the National Council on Education and the Disciplines, has described QL as a cultural field where language and quantitative constructs merge and are no longer one or the other, reflecting the continued suffusion of arithmetic with meanings from societal contexts.
The two mathematics embodied in Hardy’s real and useful components and in Cohen’s sophisticated mathematics and commercial arithmetic are represented today by formal school and college mathematics (formal, for short), on the one hand, and what I will call QL mathematics, on the other. The influence of both Hardy’s real mathematicians and long traditions of content has led to a mathematics curriculum that is dominated in grades 8-14 by a hurried and linear sequence of geometry, algebra, trigonometry, and calculus (GATC, for short). Generally ineffective in educating for QL, this GATC sequence is underwritten by the perceived educational needs of future scientists, engineers, and mathematicians, who comprise approximately one-fourth of the college population. With QL mathematics becoming both more demanding and more in demand, the pressure on formal mathematics to respond with more effective QL education is increasing. However, in addition to the enormous inertia in K-16 mathematics, several worries generate arguments against significant systemic changes in curriculum or pedagogy in service to QL.

Some problems in educating for QL result from our repeating mistakes made in colonial America. We have maintained the circumstance of two mathematics by keeping distance between QL mathematics and formal mathematics. We have stripped the GATC sequence to essentials, cutting the material into incoherent bits that are difficult to learn, much less to use. We even misuse the twenty-first century analog of ready reckoners—handheld calculators—as a total substitute for doing and understanding arithmetic and algebra.

Today, formal mathematics is institutionalized in American society, and QL mathematics, which should be the darling of practical-minded legislators, struggles for a toehold. The GATC sequence is firmly embedded in local, state, and national legislation via assessment requirements for K-12 progress, school-to-college transitions, and college degree requirements. The inclusion of so-called contextualized items on these assessments has met with worries that they do not represent the intended mathematics.

Beyond these circumstantial impediments to QL education, there is legitimate concern about the commingling of formal and QL mathematics—and, in particular, about the consequences for formal mathematics. Learning mathematics for long-term transfer is known to be difficult to achieve, especially for transfer to the multiplicity of unforeseen contexts required by QL. The self-serving nature of higher education disciplines and the relative insularity of mathematics mitigate against major changes to address this educational challenge. Below I explore four more specific aspects of the arguments deployed against major changes in service to QL.

"QL Is Difficult"

There can be no doubt that QL is difficult. Ironically, much of the mathematics involved is relatively elementary; it largely comprises middle-school arithmetic. This leads many to conclude that QL is a K-12 issue rather than a collegiate issue. The difficulty of QL, however, is rooted in its sophisticated uses of elementary mathematics and their immersion in extraneous, varied, and possibly confusing terminology. Using mathematics in multiple and unpredictable contexts requires both an understanding of mathematical concepts and practice at retrieving and applying them. Often, contexts are replete with the language of science, statistics, economics, or engineering. Relevant information may be ambiguous or hidden. Sorting all this out, modeling with mathematics or statistics, doing the mathematics, and interpreting the results is challenging indeed.
The dominant pedagogy in formal mathematics encourages students to learn abstract concepts before learning to apply them. The reverse would be QL-friendlier: looking at multiple applications and extracting the abstractions, an approach more common in business and engineering pedagogy. Learning mathematics by this QL-friendly pedagogy takes only one step toward QL, however; the mathematics must be practiced in multiple contexts to develop the habits necessary to recognize when to use it. Only part of this practice can be achieved in mathematics courses, and not all that is necessary can be achieved in formal schooling. But some of it must be achieved in mathematics courses and more in formal schooling, and schooling should prepare students to continue to practice retrieving and applying the mathematics in contexts beyond school. QL is a habit of mind, and habits are developed from repeated practice.

“QL Cannot Be Taught”
The claim that QL cannot be taught is subject to two interpretations: either it is simply impossible to teach QL, or QL cannot be taught in the current educational environment by using prevailing pedagogical practices.

Indeed, K-12 teachers and college mathematics faculty are not prepared to educate effectively for QL. Many college mathematics faculty, including this author, were educated in formal mathematics only. In other words, ours was what Alan Schoenfeld (2001) has described as an “impoverished education” that excluded real-world concerns. We have little practice in deriving abstract mathematics from multiple applications; we have years of practice in demonstrating largely artificial applications of abstract mathematics.

Not much has changed since Alan Schoenfeld and I were educated. School and college mathematics remains narrow, hurrying toward the crown of calculus. QL requires a broader curriculum with more attention to contextual problems.

Is QL impossible to teach? To some extent, perhaps; but it is no more difficult to teach than any other habit of mind—creative writing, reasoning, problem solving, or critical thinking. As we know, developing habits of mind requires practice in a variety of contexts. Mathematics alone cannot teach QL, but, because of its privileged place in K-12 and college curricula, it clearly must be a contributor. The better coordination of school and college curricula required to develop QL will benefit other competencies as well. The college major’s emphasis on study in depth has served American education well for the past century. In fact, the accomplishments of the graduates of this system have created the need for more synergism in education. Yet, because academic disciplines are responsible for the need for differently educated citizens, it behooves them to address the consequences of their work.

“Making Major Changes in American Education Is Unrealistic”
The history of American education, particularly higher education, certainly supports the claim that making major changes is unrealistic. The only major change in American higher education occurred about a century ago with the move from the classical curriculum to the system of majors, electives, and general education courses. Even during this paradigm shift, mathematics remained the same; it continued to offer the same courses it had previously, whether for general education or for preparation for in-depth study in mathematics or some mathematics-intensive major. There are reasons beyond QL for reform in collegiate mathematics—a very different college population, a vastly different society, and remarkable new technologies, to name a few—and accountability pressures are mounting. Major changes in collegiate mathematics are likely.

Recent changes to the way writing is taught in college offer the only model for widespread and lasting interdisciplinary cooperation. Although the success of writing across the curriculum and writing in the disciplines is limited, it nonetheless demonstrates the potential for the kind of interdisciplinary collaboration QL also requires. If mathematics faculty can coach writing, then literature faculty can coach QL.

“Emphasizing QL Will Harm Mathematics”
The most challenging and serious obstacle to implementing major educational changes to achieve QL is the fear of harming mathematics. Many believe that more rigorous education in mathematics will solve the QL problem, and they do not want to encourage schools and colleges to flee the rigors of Euclid and Euler (geom-
They fear that teaching contextualized mathematics will water down the mathematics, that far fewer students will learn the formal mathematics required for science and engineering. Couple this worry with the lack of understanding of QL and how to achieve it, and you have a legitimate concern.

The content of mainstream formal mathematics undergirds the education of scientists, engineers, and mathematicians. If this is unsuited to education for QL, then will an altered QL-friendly stream be unsuited to the education of scientists and engineers? We need a curriculum that meets both needs, one that is accessible at different points and from different backgrounds. Failing that, we would need to make choices at some point to split—or partially split—the stream. So far, we have been unable to fashion a split in such a way that one can move from one branch to the other without large loss of ground. We have also been unable to dispel the perception that QL-friendly tracks—general education tracks—are lower-status tracks for lower-class students. This has prompted equity advocates to favor the historically higher status GATC track—the college preparatory track in schools—for everyone.

Why trade knowledge of an ancient discipline with centuries of contributions to society for something we barely understand and have little evidence we can actually teach? The major reason is that so much is at stake. As Carnevale and Desrochers point out (2003, 29), “the wall of ignorance between those who are matematically and scientifically literate and those who are not can threaten democratic cultures.” And as the consequences of uninformned participation in an increasingly complex American democracy and economy become direr, QL will become a more prominent civil right—not because of historical educational practice, but because QL is essential for the American guarantees of life, liberty, and the pursuit of happiness.

**Conclusion**

Adequately addressing education for QL requires changes in both curriculum and pedagogy. Broad support for changes will require well-understood standards and assessments for QL as well as success stories about educational models that work. Efforts in this direction are in progress and should serve to better define QL and how it can be achieved developmentally.

The major curricular change involves bringing the two mathematics—formal and QL mathematics—closer together through more contextual teaching, thereby making mathematics more apparently relevant to contemporary society. At the same time, other disciplines must help through a coordinated integration of QL concepts across the curriculum.

Needed pedagogical changes, admittedly more difficult, include increased extraction of abstractions from examples, better understanding of effective contextual teaching practices, and more attention to research results about how people learn. For example, research results show that students need to learn how they learn in order to extend learning beyond school. Making education more connected to society should promote such learning.

Can we succeed at improving QL without doing harm to mathematics? In a word, yes. Undergraduate mathematics desperately needs refocusing, and becoming more immediately relevant can only make it both more effective and more appreciated. The mathematics curriculum would profit from a good weeding, making room for more contextual teaching. Interdisciplinary cooperation can significantly strengthen mathematics and the other disciplines, and the pedagogical changes required are long overdue. Besides, as noted earlier, this need for a high level of QL is rooted in the success of American education, which, therefore, bears responsibility for addressing the consequences.

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In the fall of 1998, Hollins University undertook a major overhaul of its general education program. The new requirements, approved in 2001, allow for foundational work in selected skills areas (writing, oral communication, quantitative reasoning, and information technology) as needed, and then encourage the melding of skills acquisition within courses designed to introduce students to various perspectives (for example, ancient/modern, creative, global, diversity). This new general education program includes two quantitative reasoning (QR) requirements: a basic skills requirement and an applied skills requirement.

The QR basic skills requirement is designed to help students gain an understanding of the fundamental mathematical skills required for success in quantitative reasoning. This requirement can be satisfied by achieving a satisfactory score on the “Quantitative Reasoning Assessment” (given to new students each fall) or by successful completion of “Mathematics 100: An Introduction to Quantitative Reasoning.” A student who has satisfied the QR basic skills requirement will demonstrate a baseline understanding of such topics as algebra, graphing, geometry, data analysis, and linearity. The goals of the QR basic skills requirement are

- to ensure that students understand mathematical and statistical reasoning;
- to ensure that they know how to use appropriate mathematical and/or statistical tools in summarizing data, making predictions, and establishing cause-and-effect relationships.

In addition to covering what may be considered to be traditional topics in the mathematics curriculum, our introductory QR course has a weekly computer lab. Students learn how to use Excel in a variety of ways—to graph and analyze data, to explore and model growth, and to learn about loans and financial planning.

The QR applied skills requirement is designed to provide students with the opportunity to apply quantitative skills as they solve problems in fields of study in which they have an interest. This requirement can be satisfied by passing a course designated as a QR applied course. The goals of the QR applied skills requirement are

- to give students the opportunity to apply mathematical and statistical reasoning in a chosen discipline;
- to involve students in the application of quantitative skills to problems that arise naturally in the discipline in a way that advances the goals of the course and is not merely a rote application of a procedure.

Each QR applied course must include at least two QR projects. For example, a project might include data collection, discussion of the data, collaborative work on finding appropriate uses of the data, use of appropriate technology in presentation and writing. The end result of each QR project should be a written assignment that includes a statement of the problem, an explanation of the methods used, and a summary of the results. When appropriate, the written assignment should discuss any limitations encountered and possible

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*This article is adapted from Caren L. Diefenderfer, Ruth Alden Doan, and Christina A. Salowey, Interdisciplinary Quantitative Reasoning, Innovation and Curriculum Development at Hollins University. Monograph Series, no. 3 (Hollins University, 2004).
improvements to the procedure and/or results.

Faculty Development
The quantitative reasoning program got a major boost from an NSF grant for “A Faculty Development Program for Quantitative Reasoning Across the Curriculum,” which brought four visiting scholars to campus during 2000-2001. Each scholar gave an evening lecture to the university community and led a faculty workshop to explain how they approached quantitative methods in a variety of disciplines at their institutions.

Hollins Professors Diefenderfer and Hammer led two series of four-day workshops where faculty members discussed the recently published *Mathematics and Democracy*, investigated topics in Hollins’s basic QR course (“An Introduction to Quantitative Reasoning”), shared and critiqued one another’s QR project ideas, and presented their QR work in progress. Thus, instructors had the opportunity to test their assignments on a willing audience and to receive feedback on their proposed projects. Twenty faculty members participated in the NSF-funded workshops, and their work resulted in twenty-seven QR applied skills courses.

Pedagogy
The implementation of an applied quantitative reasoning requirement in the general education program at Hollins encouraged professors from many different disciplines to recognize that quantitative reasoning is a skill necessary for the mastery of material in courses they are already teaching. Faculty who teach quantitative reasoning material in their courses observe benefits both in terms of their development as more active, engaged, and creative teachers and in their students’ classroom experience.

The NSF-funded seminars were a productive first step for many professors. The introductory readings and interactions with the organizers gave the participants ideas about changing their own classes to include quantitative reasoning assignments. The seminars also provided a template for writing a quantitative reasoning assignment. For faculty who teach in fields other than mathematics and the sciences, the creation of an exercise that guides the students step-by-step through a problem-solving process may not be a regular part of their classroom preparation. The peer review of the quantitative reasoning assignments proved instructive for all participants and added to their pedagogical repertoire. The gathering of faculty from several different disciplines had the additional benefit of increasing collegiality among those faculty, generating interest in the goals of courses taught outside of their own disciplines, and raising the level of respect they had for each other’s intellect, scholarly ability, and pedagogical technique. The seminars thus expanded the pedagogical horizons of the faculty involved and created a mini-community of exploration and learning that bolsters the educational mission of the university.

Many faculty noted that the techniques they used to present the quantitative reasoning assignments created new classroom dynamics. A number of the assignments were multi-step procedures involving data collection, calculation, presentation of the results, and interpretation. The step-by-step nature of the assignments alleviated student anxiety about the quantitative component, made the assignments accessible, and had a natural fit in the course material. Group work, especially for data collection, was a common feature, and faculty found that students enjoyed working with each other. Students discovered that they each had different strengths that contributed to solving a complex problem. Student collaboration added a strong element of fun and excitement to the task, thus removing any stigma that the quantitative reasoning course label might have carried. Instead of being daunted by the in-depth problem solving they were being asked to do, students became engrossed in the work, drawn in by its practical nature, and were proud of the results they were able to produce.

The quantitative reasoning exercises developed by many faculty members have the added benefit of coming from real-life examples and data. In a classics class, for example, the measurements of extant Doric temples raise many open-ended questions about temples from antiquity: was Vitruvius’s ratio actually the standard, or was it only his imagined ideal? In addition, the project on the ratio of column height to column diamet-
ter shows clearly that professionals need to find and analyze more data from the time period of Vitruvius. In a history class, as students investigate the extraordinary document that records quantitative information about Napoleon’s march to Moscow, their analysis conveys a seriousness and weight that make this historic event become real, present, and disastrous. In each of these settings, academic material comes to life when dealing with the messy, complicated, sometimes incomplete real-life data. We have discovered a powerful principle: using data from real situations invites students to explore and even struggle to understand the material so that they own the process of interpretation. Contrived textbook examples are simple and easy, but they do little to build confidence or promote deep understanding.

Assessment

We have used several assessment instruments to measure the effectiveness of the Hollins program since we introduced our basic skills quantitative reasoning requirement in the fall of 1998. One measure of effectiveness is to consider final course grades. Of the 349 students who completed “Mathematics 100” during the past five years, 88.5 percent passed the course.

All entering students complete a quantitative reasoning assessment, and students enrolled in the basic skills quantitative reasoning class retake the assessment at the end of the semester. A comparison of these two scores gives us another measure of their progress. Our data from this five-year period show that, on average, students improve 13 percent on the assessment after completing the course. Both of these numerical measures indicate that students who enroll in the course become stronger in analyzing quantitative information.

We have also collected qualitative data. During 2001-2, a campus assessment committee devised student perception surveys to assess the new general education requirements. The data for students in the basic skills quantitative reasoning class show a very high self-assessment of improvement. The percent changes range from +93.9 percent to +140.7 percent on four survey items. Data for students in the applied skills quantitative reasoning classes also show positive student self-assessment, with percent changes of 51 percent to 52 percent.

Hollins has been selected as a pilot site for another NSF grant, “The Development of Assessment Instruments for the Study of Quantitative Literacy.” The purpose of this grant is to develop appropriate assessment instruments for quantitative literacy and to train a small group of professionals to conduct the assessment. Hollins faculty members will help to write the assessment instruments and then test the instruments on campus. This work will occur during 2004.

Conclusion

Providing all students with the knowledge, ability, and confidence to solve and understand quantitative issues was the original goal of the Hollins quantitative reasoning program. In designing projects to give students these skills, faculty members have become more conscious and deliberate about creating assignments in all their classes. We have discovered that quantitative perspectives can be found everywhere, not just in the classes that now satisfy our applied quantitative reasoning requirement.

We have discovered that quantitative perspectives can be found everywhere, in many disciplines and in many classes, not just in the classes that now satisfy our applied quantitative reasoning requirement.
Statistical literacy is critical thinking about arguments that use statistics as evidence. Statistical literacy focuses primarily on inductive reasoning and strength of argument for a disputable claim. In observational studies, statistics are contextual; they depend on what is taken into account. This focus on context is why statistical literacy can be viewed as an essential component of a liberal education. And given the role of observationally-based statistics in modern life (e.g., public policy issues, school quality, health choices), statistical literacy should be an essential component of education for responsible citizenship and effective personal decision making.

According to AAC&U’s Presidents’ Campaign for the Advancement of Liberal Learning, the aims of a liberal education include “developing intellectual and ethical judgment, and expanding cultural, societal and scientific horizons.” If this education is to be a twenty-first century liberal education, then students must be statistically literate.

Statistical literacy includes many elements of quantitative literacy. It involves the mathematical approach in focusing more on signal and pattern than on noise or chance; it involves the statistical approach in focusing on the role of context, conditional reasoning, and variation. But statistical literacy goes beyond quantitative literacy or numeracy by focusing on the ability to read, to interpret, and to communicate. Numeracy focuses primarily on numbers; statistical literacy focuses more on the words framing the numbers.

The primary goal of the W. M. Keck Statistical Literacy Project at Augsburg College is to develop statistical literacy as an interdisciplinary curriculum in the liberal arts. William Frame, president of the college, observed, “Augsburg has a long history of uniting the liberal and the practical to prepare students as citizens and stewards of the world. This concept [statistical literacy] is central to our effort in behalf of interdisciplinarity.” And indeed, statistical literacy is a key component of a practical liberal arts education. As Augsburg’s director of general education, Professor Joan Griffin, has noted, “statistical literacy, with its strong focus on arguments involving statistics in public policy, helps students become better citizens and leaders.”

**Numbers and Words**

In the statistical literacy course at Augsburg College, students learn to distinguish the truth of the statistic from the strength it gives in an argument. One example involves two hunters being chased by a bear (Friedman 1997). The first hunter says, “We’re doomed. This bear can run faster than we can.” The second hunter says, “No! I don’t have to outrun the bear; I just have to outrun you.” The truth of the statistic (run faster) is independent of its support for the truth of the claim.

By focusing on grammar to distinguish association from causation, students reflect on differences between statements like the following: “People who are heavier tend to be taller,” “as weight increases, height tends to increase,” “increasing weight tends to increase height,” and “if people gain weight they can expect to become taller.” The first asserts association; the last asserts causation. To social scientists the middle two assert association, but to most readers they assert causation.

Students also focus on the definitions of soft terms such as “bullying.” What is it? Can “bullying” be defined to make...
Students quickly learn that small changes in syntax can produce large changes in semantics. In describing ratios, they learn that “the percentage of women who are runners” is different from “the percentage of women among runners.” In comparing ratios, they learn that “widows are more likely among suicides than are widowers” is quite different from “widows are more likely to commit suicide than are widowers.” They describe and compare rates and percentages in tables and graphs. These students are studying conditional reasoning using ordinary language instead of algebra.

Moreover, this statistical literacy course teaches students to be aware that statistics can be ambiguous. Compared to Hawaii’s rate, for example, the 1996 auto death rate in Arkansas was 104 percent higher (per vehicle), 78 percent higher (per registered driver), and yet 77 percent lower (per mile of road). The term “auto death rate” is ambiguous; it does not say whether the rate is per vehicle, per driver, or per mile of road. Students also study the “confusion of the inverse.” They learn that “most accidents occur within twenty five miles of home” does not mean “accidents are less likely when driving further from home.”

Numbers in Context
Having learned how to use words more precisely, students have the foundation in conditional reasoning needed to see that, in observational studies, statistical associations are contextual: their value depends on what one takes into account.

Students are generally aware that taking into account the size of a group can change the direction of an association involving counts. They know that even if “more people are unemployed in California than in Iowa,” it still may be that “the unemployment rate is higher in Iowa than in California.” But they lack experience in seeing how ratios such as averages or percentages can change by taking into account the influence of a confounder. A confounder is any related factor or lurking variable that is tangled up with an association.

For example, students are often perplexed to learn that the hospital with the highest death rate in a state is often the leading research hospital. This seems to be a basis for action, but students realize this may not mean the research hospital is a bad hospital, i.e. that it has untrained doctors, inept nurses, and inadequate facilities. They quickly realize the high patient death rate may be due to a confounder: the high percentage of patients who are in poor condition. Students realize they must think—or rethink—about context before concluding or acting.

This statistical literacy course helps students learn a new graphical technique for standardizing patient death rates so that two hospitals having different mixes of patient conditions can be compared on the same basis. Using this technique, they can calculate weighted averages graphically and they can readily explain how or why a statistic changed. These students are handling problems in multivariate thinking that are not taught in most introductory statistics courses.

Using this graphical technique and summary data from the U.S. Statistical

In addition to its annual meeting, AAC&U offers a series of working conferences and institutes each year. Additional information about the upcoming meetings listed below is available online at www.aacu.org/meetings.

Network for Academic Renewal Meetings
October 21–23, 2004
Diversity and Learning:
Democracy’s Compelling Interest
Nashville, Tennessee

November 11–13, 2004
Educating Intentional Learners: New Connections for
Academic and Student Affairs
Philadelphia, Pennsylvania

February 17–19, 2005
General Education and Assessment: Creating
Shared Responsibility for
Learning Across the Curriculum
Atlanta, Georgia

April 14–16, 2005
Pedagogies of Engagement: Deepening
Learning In and Across the Disciplines
Bethesda, Maryland

2005 Annual Meeting
January 26–29, 2005
Liberal Education and the New Academy: Raising
Expectations, Keeping Promises
San Francisco, California
Abstract, they see that 65 percent of the $16,000 black-white family income gap is explained by the difference in family structure (the percentage of families who are headed by a married couple). Students can also use this graphical technique to see that the 2000 National Assessment for Educational Progress fourth grade math scores for Utah and Oklahoma would reverse after taking into account the influence of family income (based on the federal criteria for lunch subsidies). Students learn that a reversal of an association (Simpson’s Paradox) reflects the influence of a confounder in observational data.

Throughout this course students critically reflect on everyday claims in the news that use statistics as evidence: “people who drink green tea weigh less”; “kids who eat a good breakfast do better in school”; “kids who skip class do worse”; etc. Students look for confounders that could provide alternate explanations for these observed associations. Since this critical thinking focus helps them make sense of things encountered in daily life, students find this part of the course especially challenging and rewarding.

**Statistical Literacy and Liberal Education**

As the Association of American Colleges and Universities points out in its “Statement on Liberal Learning,” a liberal education involves “the capacity to understand ideas and issues in context.” Thus, statistical literacy, critical thinking about statistics as evidence, is an integral component of a liberal education since a key goal of statistical literacy is helping students understand that statistical associations in observational studies are contextual: their numeric value and meaning depends on what is taken into account. The need to deal with context and confounding is ubiquitous to all observational studies, whether in business, the physical sciences (e.g., astrophysics), the social sciences, or the humanities.

Is statistical literacy important? The students think so! On the first day of class, most students say they would not take the course unless it satisfied a graduation requirement. On the last day of class, students were asked if this course should be required for all students. On a five-level scale, 20 percent said “absolutely” while another 30 percent “strongly agreed.” This is a big change.

Should statistical literacy be taught across the curriculum rather than as a single course within a department? This would seem ideal provided it were sustained and done well. But as Joel Best (2004) notes, “the lack of a departmental owner meant that teaching critical thinking remained everyone’s responsibility—and therefore no one’s.” He concluded that without a disciplinary home, general skills courses such as critical thinking and statistical literacy may not survive since “statistical literacy falls between the stools on which academic departments perch.” This is a serious and seemingly intractable problem for these interdisciplinary service courses.

Departments tend to teach service courses as the first course for majors rather than as the last course for non-majors. Disciplines may support the idea of broadening a service course only if they don’t give up anything from their discipline in the process. For example, a group of international statistical educators studied statistical literacy in depth. When they were asked if the introductory statistics course should focus more on the role of context and confounding in observational studies they were generally supportive. But when asked if this should be done even if it meant reducing course content involving statistical inference, this same group was seriously divided.

It appears that a much larger effort will be required to locate and nurture a suitable home for those general methods courses that develop the core capacities of a twenty-first century liberal education.

**References**


**STATISTICAL LITERACY ON THE WEB**

- [www.StatLit.org](http://www.StatLit.org) is a key site for articles, books, and links on statistical literacy.
- [www.Augsburg.edu/StatLit](http://www.Augsburg.edu/StatLit) is home for the W. M. Keck Statistical Literacy Project at Augsburg College.
Improving the quantitative skills of all students and enhancing the development of quantitative skills within major programs require looking and working well beyond the boundaries of individual departments. At James Madison University (JMU), the Department of Mathematics and Statistics has been reaching out and connecting with general education and through departments and programs across campus with a comprehensive approach. Careful thought and planning is required to achieve a balanced and comprehensive approach to improving quantitative skills and reasoning in response to multiple demands. These sometimes competing demands include the need to establish effective and measurable liberal education goals for all students, the need for specific skills for students in traditionally quantitatively-oriented disciplines, the evolving need for quantitative skills in areas not previously thought of as “mathematical,” and very special needs for teacher preparation. One guiding principle is to encourage more students to learn more mathematics and statistics and to provide more opportunities for them to apply this knowledge within major programs.

**Freshman Advising**

All entering freshmen at JMU take an algebra-based placement exam. We have carefully examined the mathematics placement scores and the eventual performance of students in basic mathematics and statistics courses, using the information to develop a matrix for freshmen advisors in collaboration with the general education program. Our intent is to improve placement and, therefore, success in mathematics and statistics courses. Standards are not changed, but we try to avoid advising students into courses where they have little chance of succeeding. We also try to avoid “math-phobic” advising—the kind of advice that would suggest a student go take that one required math course and get it over with—or advising that suggests that freshmen in certain majors must take a certain set of courses immediately, even if they are not fully prepared.

We have extended the attention to advising to combine mathematics placement scores in other areas. For example, mathematics placement and math SAT scores are reliable predictors of passing general chemistry. Student effort is obviously an important factor, but below certain placement scores the probability of passing is essentially zero due to the extensive use of algebraic manipulations. As a result, some entering biology majors may be advised to delay general chemistry until they have completed a particular mathematics course. Similarly, we have found that students who might have mathematics placement scores adequate to enroll in statistics do not perform well if they have lower SAT verbal scores. However, after taking a course in writing, their chances of succeeding in statistics improve because of increased proficiency in working through written problems.

**Support**

Recognizing that nearly all students can run into a mental block in a mathematics course and may benefit from additional support, we have developed the
Science and Mathematics Learning Center to provide professional assistance in working through problems in basic mathematics, statistics, and science courses. The center has two full-time professional faculty members who, assisted by trained students, work with approximately 10,000 student visitors each year. Assessment of grades for students using the center indicates improved performance. Similarly, we provide supplemental instruction by student mentors for a number of key classes in which students often struggle. This too produces positive results.

Paying careful attention to hiring, staffing, and delivery of courses is essential. We search for faculty who are balanced teachers and scholars—and who are genuinely interested in working with undergraduate students. Departments match faculty strength and pedagogical approaches to the levels of the course. For example, peer mentoring has produced benefits in general chemistry and in entry-level computer information system courses, both of which emphasize problem solving.

Assessment
All entering students take assessment exams during orientation for the fall semester. In the spring semester of the sophomore year, the students are reexamined during a day devoted to assessment university-wide. Thus, we can examine their performance and compare it with the courses they have taken to satisfy general education requirements. The exams were developed by our Center for Assessment and Research Studies in conjunction with faculty members who have written questions related to the learning objectives for general education. One of these exams covers quantitative literacy. In addition to the testing of all students, the Department of Mathematics and Statistics is in the process of assessing outcomes across a series of mathematics courses as part of a project with the Mathematical Association of America.

Curricular Changes
Within the Department of Mathematics and Statistics, there have been many curricular changes designed to improve the preparation of students. For example, a new calculus sequence was developed for students who might have gaps in their preparation and not be fully prepared for science-oriented Calculus I. The two-semester course combines calculus and precalculus, is very rigorous, and fully prepares students to be successful in Calculus II. This route has become very popular for many majors in chemistry and biology. Previously, many of these students would have enrolled in a softer “terminal” calculus course, making it difficult for them to continue on to the advanced mathematics courses that are becoming increasingly important to these disciplines.

A course in discrete mathematics was expanded into a sequence in very close collaboration with the computer science program. A course in quantitative geology has been offered as a result of collaborations of geologists and mathematicians. Working with the College of Education, an elementary and middle school teacher preparation program has been developed, which includes three core courses for all teachers followed by four upper-division courses for middle school mathematics teachers and elementary school mathematics specialists. A computational science minor has been implemented jointly with physics and a course in “Mathematical Models in Biology” has been developed and co-taught. Each of these areas grew from faculty collaborations, and new faculty have been hired to support the developing connections across disciplines. We also are seeing evidence of additional quantitative and analysis courses springing up within the disciplines.

Minors and Majors
Students can benefit greatly from a minor. We offer minors in mathematics and in statistics, and both are increasingly popular. The mathematics minor is often taken by students in the natural sciences and in business. The statistics minor is frequently chosen by students in the health professions, psychology, and sociology. For the students in the biomathematics course mentioned above, the mathematics majors
indicated that they would take more biology and the biology students that they would take more mathematics, with some deciding to add a minor. These impacts suggest that infusing more mathematics into biology (and other courses) may be successful.

As students become more aware of the practical benefits of additional mathematics and the interesting and rewarding new opportunities that result, faculty in mathematics and statistics are collaborating with those in science, business, and computer science on plans to make it more convenient for students to pursue a double major while still meeting the core demands of the individual disciplines. A new statistics major promises to be a popular choice as a second major for students in several other disciplines.

**Interdisciplinarity**

Many fields have become more quantitative over time, especially biology, chemistry, and finance. This is also true of professional programs and a wide range of disciplines. But what quantitative applications and approaches are being used? Have they changed over time? What might be the best preparation for students?

These questions are not simple ones and require connections and conversations among mathematicians, statisticians, and faculty working in other programs. Our experience suggests that collaborative course development and research help faculty appreciate quantitative applications in other fields, which is especially true in interdisciplinary research questions. But that is only one piece of the answer to these questions. Making substantial progress in improving the quantitative skills and reasoning of students requires careful attention to the development of a student in a very intentional way—something most majors rarely consider. For example, if we are interested in having students engage in a capstone experience that involves research, possibly as part of an interdisciplinary team, we should align a curriculum intentionally to prepare students for such an experience. Sometimes majors have been constructed and offered as collections of courses, with coherence often lacking. However, if we are to develop quantitative reasoning in students, it cannot be accomplished solely by requiring certain courses in mathematics or statistics; it will require attention to enhancing quantitative skills within a major program.

Undergraduate research experiences have brought faculty together in new ways and have developed further connections between disciplines. For example, our Materials Science Research Experiences for Undergraduates program, funded by the National Science Foundation, includes faculty mentors from several departments, including mathematicians, and mathematics majors have participated in research programs in other disciplines. Statisticians frequently work with students from a range of disciplines as they engage in senior projects or other research experiences. More recently, work in visualization emanating from the Center for Computational Mathematics and Modeling by applied mathematicians and physicists is sparking interest from faculty from a wide range of areas, including economics, art, computer science, and geology, among others.

**Conclusion**

Rapid developments in many fields, emerging disciplines, blurring boundaries, and the need for enhanced quantitative skills challenge us all to provide improved quantitative preparation for our students. Responding effectively requires careful reexamination and coordinated design of curricula, which may in some cases mean changes in cognate requirements, revisions to existing courses, and more attention to building quantitative skills within major programs. We suggest that, in order to be successful, it will be necessary to recognize student backgrounds and support developmental progress of students in an organized way, to reach out from mathematics to disciplines to understand and serve their needs, to build connections with disciplines, to use multiple approaches and curricula designed for different purposes, and to assess outcomes related to learning goals.

Clearly we are calling for a comprehensive approach and fully recognize that many steps are required over a period of years to produce the kind of transformation required. What we envision is not the isolated island of mathematics that, unfortunately, has been in the minds of many faculty (or even within departments of mathematics), but rather departments at the center of a university with helpful hands interdigitating across a campus with vibrancy and excitement in pursuing quantitative solutions and applications in a wide range of settings.
The Problem

Although almost 90 percent of eighth graders expect to participate in some form of postsecondary education\(^1\) and nearly two-thirds of parents consider college a necessity for their children,\(^2\) our education system sends a confusing set of signals to students about how they can reach that goal. High school students earn grades that cannot be compared from school to school and often are based as much on effort as on the actual mastery of academic content. They take state- and locally mandated tests that may count toward graduation, but very often do not. College-bound students take national admissions exams that may not align with the high school curriculum the students have been taught. If they reach college, students face an assortment of placement tests unrelated to any of the tests they have taken already, and these tests vary from campus to campus, even within a single college system. This confusing array of exams diminishes the potential value of standards-based high school exit assessments, even in the minority of states where they currently count as graduation requirements.

The troubling result is that far too many young Americans are graduating from high school without the skills and knowledge they need to succeed. …

The Solution

What will it take to restore value to the American high school diploma? First, state policy makers need to anchor high school graduation requirements and assessments to the standards of the real world: to the knowledge and skills that colleges and employers actually expect if young people are to succeed in their institutions. In return, colleges and employers need to start honoring and rewarding student achievement on state standards-based assessments by using these performance data in their admissions, placement, and hiring practices.

To help states get started, [the American Diploma Project (ADP)] worked closely with K-12, postsecondary, and business leaders in our five partner states (Indiana, Kentucky, Massachusetts, Nevada, and Texas) to identify the English and mathematics knowledge and skills needed for success in both college and work. We first asked leading economists to examine market projections for the most promising jobs—those that pay enough to support a small family and provide real potential for career advancement—and to pinpoint the academic knowledge and skills required for success in those occupations. We also worked closely with two-

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and four-year postsecondary leaders in the partner states to determine the prerequisite English and mathematics knowledge and skills required for success in entry-level, credit-bearing courses in English, mathematics, the sciences, the social sciences, and humanities.

The result of the research is a set of benchmarks that should serve as the anchor for every state’s system of high school standards-based assessments and graduation requirements. States also can use the benchmarks to map back through the earlier grade levels to refine the standards, assessments, and proficiency levels in English and mathematics required by the federal No Child Left Behind Act (NCLB). As we conducted the research, we found an important convergence around the core knowledge and skills that both colleges and employers—within and beyond the ADP states—require. Students who meet these standards therefore will be prepared for success, whatever path they choose to pursue after high school.

Although high school graduation requirements are established state by state, a high school diploma should represent a common currency nationwide. Families move across state lines, students apply to colleges outside their own state and employers hire people from across the country. States owe it to their students to set expectations for high school graduates that are portable to other states. The ADP benchmarks can help make this portability a reality. States that adopt these benchmarks will have a ready and persuasive answer for students when they ask, “Why do I have to learn these things?”

The ADP benchmarks are ambitious. In mathematics, they reflect content typically taught in Algebra I, Algebra II and Geometry, as well as Data Analysis and Statistics. The English benchmarks demand strong oral and written communication skills because these skills are staples in college classrooms and most twenty-first century jobs. They also contain analytic and reasoning skills that formerly were associated with advanced or honors courses in high school. Today, however, colleges and employers agree that all high school graduates need these essential skills. …

Awarding diplomas to students who … cannot meet real-world demands will only mislead high school students about their chances for success as adults and minimize the potential of standards-based systems to ensure equity in the quality of instruction for all students.

Instead, state education and business leaders must devise strategies that build on, rather than discard, ongoing standards-based reforms; that sensibly ratchet up the rigor of standards, assessments, and course-taking requirements over time; and that blend them into a coherent system of requirements for earning a high school diploma that signifies college and workplace readiness. …

What States and Postsecondary Institutions Should Do

Anchor Academic Standards in the Real World

States should:
- Align academic standards in high school with the knowledge and skills required for college and workplace success.

Require All Students to Take a Quality College and Workplace Readiness Curriculum

States should:
- Define specific course-taking requirements in English and mathematics for high school graduation, and specify the core content for those courses.

Bridge the Gap between High School and College

States should:
- Hold postsecondary institutions accountable for the academic success of the students they admit, including student learning, persistence, and degree completion.
Postsecondary institutions should:
- Use high school assessments for college admissions and placement.
- Provide information to high schools on the academic performance of their graduates in college.

**Benchmarks**

The benchmarks, as well as the workplace tasks and postsecondary assignments that accompany them, represent a collaborative effort among K-12 educators, postsecondary faculty, and frontline managers to define a common core of fundamental literacy and numeracy—what high school graduates must know and be able to do to be fully prepared to succeed in credit-bearing college courses or in high-growth, highly skilled occupations.

The work of ADP differs in one significant respect from other praiseworthy state efforts to develop standards: it grounds its benchmarks in empirical evidence of what the postsecondary world—employers and educators—actually requires of employees and students. The innovative addition of actual workplace tasks and postsecondary assignments vividly illustrates the intellectual demand that high school students will encounter in high-performance workplaces or in credit-bearing first-year college courses. …

**Mathematics Benchmarks**

The ADP college and workplace readiness benchmarks for mathematics are organized into four strands:

**Number Sense and Numerical Operations**

Number sense is the cornerstone for mathematics in everyday life. … At the heart of the study of numbers is an appreciation of how numbers are used to represent real-world objects and their attributes. Working with numbers requires an understanding of the relationships between numbers, the magnitude of numbers and when to use which operation, as well as the ability to make reasonable estimations and mental computations.

**Algebra**

Mathematicians regularly identify sources of change, distinguish patterns in that change, and seek multiple representations—verbal, symbolic, numeric, and graphic—to express what transpires. The language of algebra provides a means of operating with these concepts at an abstract level and extending specific examples to broad generalizations.

**Geometry**

Geometry is an ancient mathematical endeavor, dating back to 300 BC and Euclid. Euclidean geometry, a milestone in the development of mathematics and other academic disciplines, is the study of points, lines, planes, and other geometric figures, resulting in a logical system that offers students a way to formulate and test hypotheses and to justify arguments in formal and informal ways. …

**Data Interpretation, Statistics, and Probability**

A free society is dependent on its citizens understanding information, evaluating claims presented as facts, detecting misrepresentations and distortions, and making sound judgments based on available data. When students learn to make predictions and develop and evaluate inferences from data, they are able to rely on data to answer such questions as “Will a college degree improve my earnings?” or “Which kinds of college degrees will give me access to the most opportunities and the highest pay?” The ability to apply basic concepts of probability is con-

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**About the American Diploma Project**

The American Diploma Project (ADP), a partnership of three national organizations and five states, is a collaborative effort to ensure that American high school students have the knowledge and skills necessary for success following graduation, whether in college, the workplace, or the armed services. Launched in 2001 with funding from The Hewlitt Foundation, the organizations—Achieve, Inc., The Education Trust, and The Fordham Foundation—have worked with Indiana, Kentucky, Massachusetts, Nevada, and Texas to build constituencies and develop policies to support a coherent K-16 system.

[www.achieve.org/achieve.nsf/AmericanDiplomaProject](http://www.achieve.org/achieve.nsf/AmericanDiplomaProject)
The world is filled with uncertainties, and probability is one way of addressing risk in daily living and reducing those uncertainties.

**A Note about Mathematical Reasoning**

The study of mathematics is an exercise in reasoning. Beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing, and “mathematicizing” a problem. As college students and employees, high school graduates will need to use mathematics in contexts quite different from the high school classroom. They will need to make judgments about what problem needs to be solved and, therefore, about which operations and procedures to apply. Woven throughout the four domains of mathematics—Number Sense and Numerical Operations; Algebra; Geometry; and Data Interpretation, Statistics, and Probability—are the following mathematical reasoning skills:

- Using inductive and deductive reasoning to arrive at valid conclusions;
- Using multiple representations (literal, symbolic, graphic) to represent problems and solutions;
- Understanding the role of definitions, proofs, and counterexamples in mathematical reasoning; constructing simple proofs;
- Recognizing when an estimate or approximation is more appropriate than an exact answer and understanding the limits on precision of approximations;
- Distinguishing relevant from irrelevant information, identifying missing information, and either finding what is needed or making appropriate estimates;
- Recognizing and using the process of mathematical modeling: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem;
- When solving problems, thinking ahead about strategy, testing ideas with special cases, trying different approaches, checking for errors and reasonableness of solutions as a regular part of routine work, and devising independent ways to verify results;
- Shifting regularly between the specific and the general, using examples to understand general ideas, and extending specific results to more general cases to gain insight.

The full benchmarks, as well as the workplace tasks and postsecondary assignments that accompany them, are available for download at www.achieve.org/achieve.nsf/AmericanDiplomaProject.

**Workplace Tasks and Postsecondary Assignments**

Workplace tasks and postsecondary assignments follow the benchmarks to illustrate their practical application beyond high school. It is important to note that the workplace tasks and postsecondary assignments are not meant to describe the quality and complexity of high school assignments. Although the benchmarks, tasks, and assignments may be used in the future to inform the development of high school lessons, the tasks and assignments included here are designed simply to illustrate the intellectual rigor of real-world environments beyond high school and the applicability of the ADP benchmarks in postsecondary and workplace settings. …
The Greater Expectations initiative focuses on important outcomes of college-level learning, outcomes that are intended more powerfully to prepare students for lives of creative and thoughtful intelligence, professional excellence, and engaged citizenship. The initiative calls for:

- Articulation of and focus on forms of learning that are widely needed in the modern world;
- A new intentionality about addressing expectations for student achievement across successive levels of learning, from school through college;
- Involvement of students in “authentic assignments,” i.e., the kinds of tasks that actually develop complex abilities while showing students how those abilities can be used with power in real contexts;
- Transparent assessments, linked to authentic assignments, that emphasize what students can do with their knowledge rather than their ability to pass standardized tests;
- Connection of desired capabilities to learning in each student’s major, so that study in the major becomes an essential vehicle not only for developing those capabilities but also for learning how to put them to use.

What do these premises imply for fostering quantitative literacy through school and college learning? Here are my proposals for educational change:

Create a public and policy dialogue about the uses of quantitative literacy.

The first change is to identify the ways in which quantitative literacies are actually used in contemporary society. But this should be more than an academic discussion; we must spark a broader public and policy dialogue about the need to recast and broaden our expectations for the quantitative literacy of the citizenry.

Identify kinds of learning.

The second change is to move beyond typologies of numeracy to a delineation of the kinds and levels of learning that need to be addressed, both in school and college, if students are actually to be held accountable for developing usable capabilities in quantitative reasoning and problem solving. Here again, the discussions should include policy and civic leaders as well as teachers and scholars.

Rethink high school mathematics.

The third change is to acknowledge the need to substantially retool the high school mathematics curriculum as well as the preparation of the teachers who provide that curriculum. High school study must lay a foundation for statistical as well as mathematical understanding. And it needs to incorporate context-rich practices that enable students to learn essential skills and discover why and for what purpose these skills matter.

*An earlier version of this article appeared in Mathematics and Democracy (Princeton, NJ: National Council on Education and the Disciplines, 2001). The views expressed are the author’s own.
Rethink college quantitative literacy requirements.
The fourth change is to recognize that, at the college level, no one course of study can realistically develop all the major kinds of quantitative literacies. We need to stop thinking that remedying our quantitative deficiencies is simply a matter of “fixing” mathematics standards and the corresponding curriculum.

Encourage alternative pathways.
Instead—the fifth change—we need to design multiple courses of study, each well structured to foster quantitative strategies used in specific kinds of professional and civic contexts. The analogy is to writing. Although all educated people need certain kinds of writing abilities, successful people actually deploy very different rhetorics depending on the context. Scientists, for example, make highly field-specific written arguments; politicians frame their written arguments in very different terms. We should allow college students to develop quantitative strengths keyed to their actual interests, even at the cost of underdeveloping other possible abilities that, realistically, they are unlikely actually to use.

Embed quantitative literacy in other fields.
The sixth change follows from the fifth. It is time to give up on the stand-alone general education mathematics requirement. The great majority of colleges and universities, whether research- or teaching-oriented, still insist that most students take such a course (usually selected from a limited menu of options) as a requirement for graduation. But very little is actually accomplished through this traditional approach to quantitative reasoning, and we must fundamentally rethink it. One promising strategy is to make field-related quantitative competence the standard, holding students accountable for evidence of developed ability to actually use quantitative reasoning in ways keyed to their major field(s) of study.

This sixth proposal may give the reader pause. Suppose the student’s field of study seems not to require quantitative abilities. What about English, the paradigmatic non-quantitative major?
The tough question is how to bring all fields into dialogue with the modern world. Even as I was majoring in history in the late 1960s, and assiduously avoiding all quantitative courses, my field was actually moving in a decidedly quantitative direction. Most fields are becoming more quantitative, reflecting trends in the world at large. All curricula must adapt to these realities. Today many history departments hold students accountable for knowledge of quantitative methods. Tomorrow (or at least in a few years) English departments, already infused with richly sociocultural concerns, must recognize and engage their students’ need for quantitative literacy as well.

Moreover, there is a discernible trend on college campuses toward minors and double majors. Colleges might insist that students choose at least one area of concentrated study, whether a major or a minor, that requires and fosters quantitative competence.

Whatever strategy we choose, we must recognize that it really is malpractice to allow students to slip through college without developing the ability to use quantitative strategies to examine significant questions. We are only shortchanging our graduates with respect to the actual demands of a numbers-infused world.

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